Determination by the BI-RME Method of Entire-Domain Basis Functions for the Analysis of Microstrip Circuits

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Abstract— **In this paper, we present a novel approach for the numerical determination of entire-domain basis functions for the analysis of shielded microstrip circuits, composed of arbitrarily shaped metallic patches with an arbitrary number of ports. This approach is based on the solution of Helmoltz and Laplace equations in the metallic areas with proper boundary condition by the Boundary Integral-Resonant Mode Expansion (BI-RME) method. Two examples validate the proposed method.**

I. Introduction

The integral equation technique is a widely used method for the analysis of boxed multilayered circuits. When applying this technique in conjunction with the Method of Moments (MoM), the choice of the basis functions used to represent the unknown electric current density is a key issue for the computational efficiency of the numerical code. Subdomain basis functions (like roof-tops on rectangular or triangular domains) have been widely used, since they are known analytically and are suitable to any shape of the metal patches. Nevertheless, a large number of subdomain functions is tipically required, thus leading to large matrix problems and reduced computation efficiency.

The advantage of using entire-domain basis functions was demonstrated, in the limited case of patches with a rectangular shape [1]. Since the electric current density is tangential to the boundary of the patches and is perpendicular to the ports, basis functions were chosen as the electric modal vectors of a waveguide with magnetic-wall condition on the boundary, and electric-wall condition on the port segments. In the case of rectangular shapes, these functions are known analytically.

Recently, the use of entire-domain basis functions was extended to the case of arbitrarily shaped patches [2], either isolated or connected to one single port. The Boundary Integral-Resonant Mode Expansion (BI-RME) method [3], [4], [5], originally developed for the determination of arbitrary waveguide modes, was used for calculating the basis functions. In fact,

This work has been supported by the Ministry of the University and Scientific Research (MIUR) under PRIN and FIRB projects.

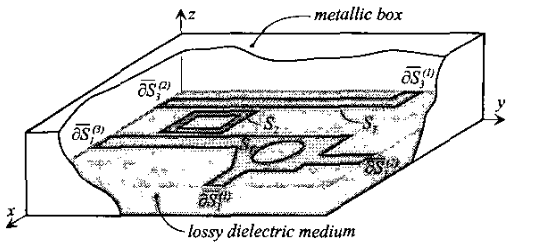

Fig. 1. Example of a shielded microstrip circuit, with arbitrarily shaped metallic areas in a lossy medium.

in the case of isolated patches, the electric modal vectors of a waveguide with magnetic walls coincide with the magnetic modal vectors of a metallic waveguide. In the case of patches connected to a single port, symmetries can be exploited to determine the basis functions, as discussed in [3]. Nevertheless, a large number of real-world components (from a simple thru line to a directional coupler) are not included in this class.

In this paper, we present a novel approach for the calculation of basis functions with mixed boundary conditions by using the BI-RME method. These functions are used in conjunction with the spectral domain method described in [2], for the analysis of boxed microstrip circuits. An outline of the approach, along with a description of the new features of the BI-RME method are described, and some numerical results are reported.

II. Novel Entire-Domain Basis Functions

The structure depicted in Fig. 1 represents a generic microstrip boxed circuit composed by different types of metallic patches connected or not to ports, with the ports represented as small gaps between the metallizations and the walls of the box (delta-gap excitations). We indicate with S_p the area of the p-th patch, with $\overline{\partial S}_p^{(i)}$ $(i = 1, \ldots, N_p)$ the part of boundary of S_p coinciding with the i-th port of the patch, and with ∂S_p the remaining part of the boundary.

For the generic patch, the unknown electric current density $\vec{J}_p$ can be represented as the sum of entire–domain basis functions $\vec{b}_r^{(p)}$, defined on the entire surface S_p. When the patch is not connected to ports, $\vec{b}_r^{(p)}$ are the magnetic modal vectors of a waveguide with a cross-section S_p bounded by electric walls on ∂S_p. Otherwise, in the case of patches with one or more ports, it is necessary to consider electric–wall conditions on ∂S_p and magnetic–wall conditions on each port segment $\overline{\partial S}_p^{(i)}$ of the patch.

For isolated and simply connected metallic areas, functions $\vec{b}_r^{(p)}$ are subdivided in two classes, solenoidal ($\vec{b}_r^{\prime(p)}$) and irrotational ($\vec{b}_r^{\prime\prime(p)}$), and can be expressed in terms of scalar potentials

$$\vec{b}_r^{\prime(p)} = -\vec{u}_z \times \frac{\nabla_T \psi_r^{\prime(p)}}{\kappa_r^{\prime(p)}} \qquad \vec{b}_r^{\prime\prime(p)} = -\frac{\nabla_T \psi_r^{\prime\prime(p)}}{\kappa_r^{\prime\prime(p)}} \quad (1)$$

In (1), $\psi_r^{\prime(p)}$ and $\psi_r^{\prime\prime(p)}$, together with $\kappa_r^{\prime(p)}$ and $\kappa_r^{\prime\prime(p)}$, are the eigensolutions of the homogeneous Helmholtz equation in the domain S_p with Dirichelet and Neumann boundary condition on ∂S_p, respectively.

When considering patches with N_p ports, $\psi_r^{\prime(p)}$ must also satisfy Neumann boundary condition on $\overline{\partial S}_p^{(i)}$, whereas $\psi_r^{\prime\prime(p)}$ must satisfy Dirichelet boundary condition on $\overline{\partial S}_p^{(i)}$. Furthermore, we must add to the set of basis functions also $N_p - 1$ harmonic functions of the type

$$\vec{b}_r^{0(p)} = -\vec{u}_z \times \nabla_T \psi_r^{0(p)} \quad (2)$$

where $\psi_r^{0(p)}$ satisfies Laplace equation in the domain S_p with Neumann boundary condition on $\overline{\partial S}_p^{(i)}$ and with proper constant values on each separate part of ∂S_p. For instance, in the case of a microstrip thru line the constant (harmonic) function tangent to ∂S_p and perpendicular to the ports is needed to represent the dominant contribution of $\vec{J}_p$ at low frequency.

More generally, in the case of M_p–times connected surfaces with N_p port segments we have to consider $M_p + N_p - 1$ harmonic functions.

III. Application of the BI-RME Method

For the determination of $\psi_r^{\prime(p)}$ and $\psi_r^{\prime\prime(p)}$ we followed the BI–RME method [3], [4], [5], which was demonstrated to be very efficient. This method is a modified boundary integral approach for the evaluation of $\psi_r^{\prime(p)}$ and $\nabla_T \psi_r^{\prime\prime(p)}$, obtained as the solution of a linear eigenvalue problem. Surface S_p (either simply or multiply connected) is considered as a part of a fictitious enlarged domain Ω_p with a rectangular shape. The eigenfunctions to be determined are defined in Ω_p, and are assumed to vanish outside S_p.

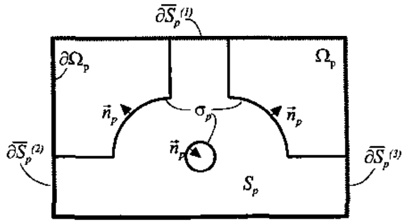

Fig. 2. Arbitrarily shaped metallic area with three ports bounded by a rectangular domain Ω_p.

The eigenfunctions $\psi_r^{\prime(p)}$ and $\nabla_T \psi_r^{\prime\prime(p)}$ are expressed as [3]

$$\begin{aligned} \psi^{\prime(p)}(\vec{r}) &= \widetilde{\Phi}(\vec{r}) + \int_{\sigma_p} G(\vec{r}, s') f(s') ds' \\ \nabla_T \psi^{\prime\prime(p)} &= \vec{u}_z \times \nabla_T \int_{\sigma_p} G(\vec{r}, s') \frac{dh(s')}{ds} ds' \\ &+ \kappa^{\prime\prime(p)2} \left(\nabla_T \widetilde{\Psi}(\vec{r}) - \int_{\sigma_p} \nabla_T \nabla_T' F_1(\vec{r}, s') \cdot \vec{n_p}\, h(s') ds' \right) \end{aligned}$$

where σ_p is the part of ∂S_p not coincident with $\partial\Omega$, s is a point on σ_p and $f(s)$, $h(s)$ are scalar unknown functions representing the discontinuity of $\partial\psi_r^{\prime(p)}/\partial n_p$ and $\psi_r^{\prime\prime(p)}$ over σ_p, respectively. The above expressions constitute the BI-RME representation of $\psi_r^{\prime(p)}$ and $\nabla_T \psi_r^{\prime\prime(p)}$ and are a combination of different terms. The integral(s) over the boundary (BI) involve quasi–static Green's functions (G and F_1) for the rectangular domain Ω_p. The remaining terms, $\widetilde{\Phi}$ and $\nabla_T \widetilde{\Psi}$, are represented as resonant mode expansions (RME) involving the modal eigenfunctions of the rectangular domain region Ω_p.

The expressions of the Green's functions G and F_1 and the resonant mode functions, reported in [3], satisfy the electric–wall condition on ∂S_p. As mentioned above, these functions allow for computing the basis functions for isolated patches or, by exploiting the symmetries (see [3]), patches with only one port.

To determine the basis functions with more than one port we need to know the expressions of G, F_1 and the resonant mode eigenfunctions for the rectangular domain Ω_p with different boundary conditions on $\partial\Omega_p$. For instance, for the circuit shown in Fig. 2, with three ports on different sides of Ω_p, it is required a rectangular domain Ω_p with electric–wall condition (black line) on the bottom side and magnetic–wall (gray lines) on the other sides.

In this work we determined the Green's functions for the domain Ω_p with mixed boundary conditions on every side of $\partial\Omega_p$. As an example, Tab. I reports the

TABLE I

Green's functions for rectangular domain with magnetic–wall condition

$$G = \frac{a}{3b} + \frac{x^2+x'^2}{2ab} - \frac{|X_0^0|+|X_0^1|}{2\pi} - \frac{1}{4\pi}\sum_{m=-\infty}^{\infty}\sum_{p,q=0}^{1}\ln T_m^{pq}$$

$$\frac{\partial^2 F_1}{\partial x \partial x'} = \frac{1}{4\pi}\sum_{m=-\infty}^{\infty}\sum_{p,q=0}^{1}(-1)^q\left[\frac{1}{2}\ln T_m^{pq} + |X_m^p| E_m^p \frac{\cos Y^q - E_m^p}{T_m^{pq}}\right]$$

$$\frac{\partial^2 F_1}{\partial x \partial y'} = -\frac{1}{4\pi}\sum_{m=-\infty}^{\infty}\sum_{p,q=0}^{1}(-1)^q X_m^p E_m^p \frac{\sin Y^q}{T_m^{pq}}$$

$$\frac{\partial^2 F_1}{\partial y \partial x'} = -\frac{1}{4\pi}\sum_{m=-\infty}^{\infty}\sum_{p,q=0}^{1}(-1)^p X_m^p E_m^p \frac{\sin Y^q}{T_m^{pq}}$$

$$\frac{\partial^2 F_1}{\partial y \partial y'} = \frac{1}{4\pi}\sum_{m=-\infty}^{\infty}\sum_{p,q=0}^{1}(-1)^p\left[\frac{1}{2}\ln T_m^{pq} - |X_m^p| E_m^p \frac{\cos Y^q - E_m^p}{T_m^{pq}}\right]$$

where: $X_m^p = \frac{\pi}{b}\left[x + (-1)^p x' - 2am\right]$ $\quad Y^q = \frac{\pi}{b}(y + (-1)^q y')$ $\quad E_m^p = \exp(-|X_m^p|)$ $\quad T_m^{pq} = 1 - 2E_m^p \cos Y^q + (E_m^p)^2$

x,y and x', y' are the coordinates of the observation and the source point, respectively

a and b are the lengths of the sides of Ω. The series converge more rapidly if $a > b$

Green's functions determined for the case of magnetic–wall condition on every side of Ω_p.

Moreover, we reformulated the BI–RME method in the case of magnetic–wall condition on all sides of Ω_p, to take into account the existence of a modal eigenfunction for the domain Ω_p with associated null eigenvalue, for the correct determination of $\vec{b}_r^{\prime(p)}$.

The possibility of mixed boundary conditions on each side of Ω_p permits also to minimize the extension of the line σ_p and the surface of the domain Ω_p. This reduces the order of the matrix eigenvalue problem associated with the determination of $\vec{b}_r^{\prime(p)}$ and $\vec{b}_r^{\prime\prime(p)}$, granting the highest efficiency to the BI–RME method for circuits with any number of ports. Fig. 3 shows some basis functions calculated with the approach presented in this paper.

The new algorithm is completed by the calculation of $\vec{b}_r^{0(p)}$, which is accomplished by solving with the MoM an integral equation obtained with a conventional BEM approach and the Green's function G used in the BI–RME method.

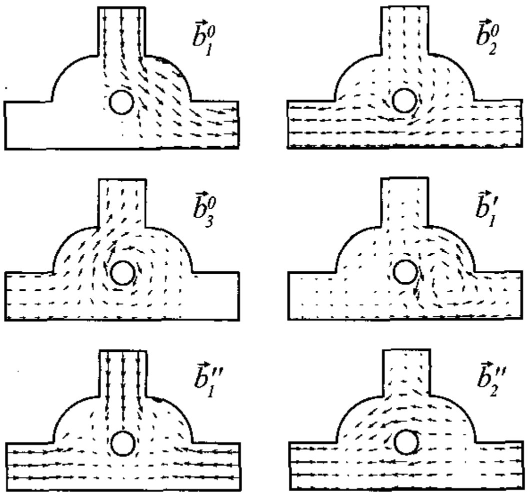

Fig. 3. Examples of some basis functions for the circuit shown in Fig. 2 calculated with the BI–RME method.

IV. Numerical and Experimental Results

For validation purposes, we present the analysis of two boxed microstrip circuits using the entire-domain basis functions.

The first circuit is a shielded microstrip single–stub filter presented in [6] and shown in Fig. 4. Fig. 5 reports the results of the BI-RME simulation compared to the measured data from [6]. A very good agreement is observed. Convergence of the results has been obtained with 77 basis functions, while in [6] the same results were obtained with some hundreds of triangular rooftop basis functions. For this example, the calculation of the basis functions takes 79 *sec* on a Pentium4 at 1.7 GHz, and the frequency–by–frequency calculation of the filter response required 3.14 *sec* per point.

The second example refers to the analysis of a bandpass filter composed of square ring resonator with a line–to–ring coupling structure (Fig. 6), firstly proposed in [7]. In Fig. 7, experimental data given in [7] are compared with results obtained by our code, showing a good agreement. The results were obtained using a total of 134 basis functions. Our code took 207 *sec* for the initial calculation of the basis functions, and 7 seconds for each subsequent point in frequency, on a Pentium4 at 1.7 GHz. The high number of basis functions depends

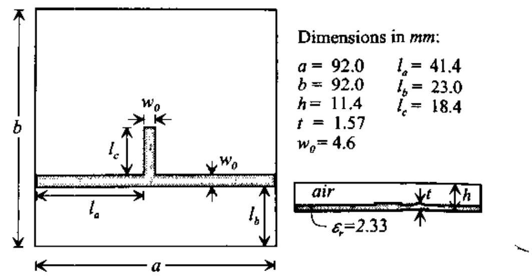

Fig. 4. Shielded microstrip single-stub filter presented in [6].

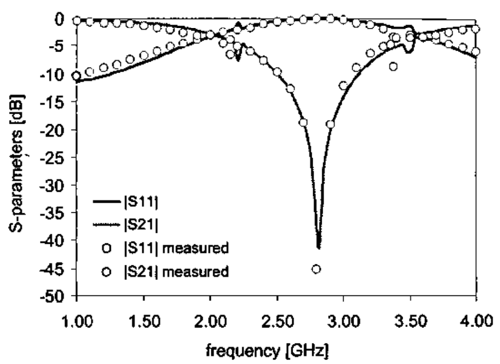

Fig. 5. Comparison between measured results presented in [6] and results obtained with the present approach, for the filter shown in Fig. 4.

on the fact that this structure is challenging due to the small gap and line width (s and w_1 in Fig. 6).

V. Conclusion

In this paper we have presented a novel approach for the calculation of basis functions with mixed boundary conditions by using the BI-RME method. This approach permits to determine the basis functions for metallic patches with an arbitrary shape and with an arbitrary number of ports, resulting in MoM matrices of small size, even in the case of complex circuits.

The analyzes of circuits of practical interest have been reported and compared with experimental data, showing the accuracy and rapidity of this approach.

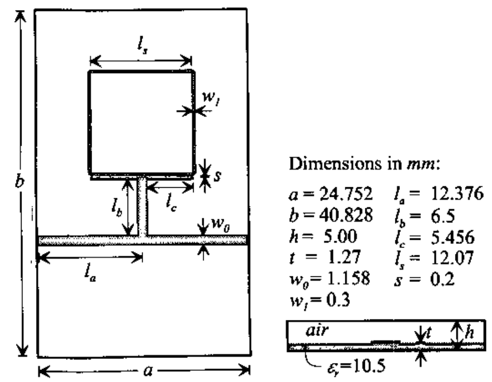

Fig. 6. Bandpass printed microstrip filter composed of square ring resonator with a line-to-ring coupling structure presented in [7].

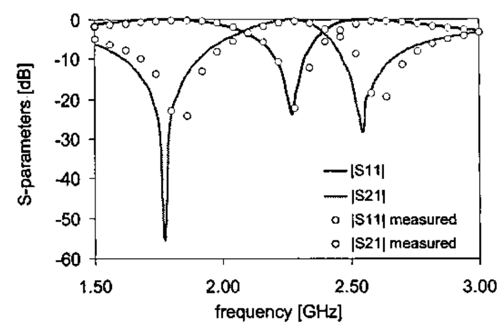

Fig. 7. Comparison between measured results presented in [7] and results obtained with the present approach, for the filter shown in Fig. 6.

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